TWO-PHASE HEAT TRANSFER IN A ONE-DIMENSIONAL CHANNEL

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The problems of heat transfer in a flow of continuous medium in a channel filled with dispersed material, and of the unsteady operation of a straight-through heat regenerator are examined; longitudinal heat transfer is ignored.

Unsteady heat-transfer (and also mass-transfer) processes between a flow of working fluid and a solid packing filling a channel are of such widespread occurrence in heat engineering that many investigations of these processes have long become classical, and their results have been given in the form of nomograms on several occasions (see, e.g., [1-4] and also [5-8]). The mathematics of the problems that arise, even when conductive heat transfer, heat transfer between the channel walls and surroundings, internal heat and mass sources, etc. are neglected, and the boundary and initial conditions are formulated in the simplest form, is so complex that the obtained solutions are usually difficult to visualize, and their analysis requires laborious numerical calculations. Hence, it is desirable to simplify the initial formulation of the mathematical problems by using approximate model considerations relating to the description of transport processes.

As was shown in [9, 10], this possibility arises when the characteristic time scale of the transport process greatly exceeds the relaxation time characterizing transfer between the flow and a single element of the packing. In this case we can convert from a system of two equations for the mean temperatures of the fluid and packing to a single approximate equation for one of these temperatures. The aim of the present work was to use such an equation to analyze some general problems of two-phase heat transfer in a straight channel in which heat is lost from the walls. For simplification we neglect longitudinal heat conduction and assume that there are no internal heat sources. We also neglect contact heat transmission through the structure formed by the packing elements. The grounds for this are obvious in the case of granular packings, packings consisting of plates oriented in the direction of the flow, etc.

The initial system of equations in dimensional variables has the form

$$\varepsilon d_0 c_0\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}\right)\tau_0=\lambda\Delta_r\tau_0-\beta(\tau_0-\tau_1),$$
$$(1-\varepsilon)d_1c_1\frac{\partial\tau_1}{\partial t}=\beta(\tau_0-\tau_1), \tag{1}$$

where Δ_r is the part of the Laplace operator containing the operations of differentiation with respect to only the transverse coordinates. We assume that the coefficients λ and β in (1) are constant.

The approximate "equivalent" equation for the mean fluid temperature τ_0 is written in the form [10]

$$\varepsilon d_0c_0u\frac{d\tau_0}{\partial x}+[\varepsilon d_0c_0+(1-\varepsilon)d_1c_1]\frac{\partial\tau_0}{\partial t}=\lambda\Delta_r\tau_0+(1-\varepsilon)d_1c_1\frac{(1-\varepsilon)d_1c_1}{\beta}\frac{\partial^2\tau_0}{\partial t^2}. \tag{2}$$

Introducing time and length scales in the longitudinal and transverse directions, and also the dimensional variables and parameters

$$t=\alpha_tT,\ x=\alpha_xX,\ \mathbf{r}=\alpha_r\mathbf{R},\ \alpha_t=\frac{(1-\varepsilon)d_1c_1}{(1+\gamma)\beta},\ \ \alpha_x=\frac{\gamma(1-\varepsilon)d_1c_1u}{(1+\gamma)^2\beta},$$

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$$\alpha_r = L,\ \gamma = \frac{\varepsilon d_0 c_0}{(1-\varepsilon) d_1 c_1},\quad \Lambda = \frac{\lambda}{(1+\gamma)^2 \beta L^2}, \tag{3}$$

we convert Eq. (2) to the form

$$\frac{\partial \tau_0}{\partial X} + \frac{\partial \tau_0}{\partial T} = \Lambda \Delta_R \tau_0 + \frac{\partial^2 \tau_0}{\partial T^2}. \tag{4}$$

A necessary and sufficient condition for applicability of this equation is fulfilment of the inequality $T \gg 1$, which is henceforth assumed to be satisfied. The packing temperature τ_1 within the framework of the approximation represented by Eq. (4) is expressed in terms of τ_0 by the relation

$$\tau_1 = \tau_0 - (1+\gamma)\frac{\partial \tau_0}{\partial T} + (1+\gamma)^2 \frac{\partial^2 \tau_0}{\partial T^2}. \tag{5}$$

We will solve Eq. (4) on condition that heat transfer takes place with the surroundings, the temperature of which is taken as the temperature zero, i.e.,

$$h\tau_0 + \partial\tau_0/\partial R_n = 0,\ \mathbf{R} \in C, \tag{6}$$

where the derivative is calculated normal to the wall and for arbitrary initial and boundary conditions

$$\tau_0 = \mu(X, R),\ T = 0,\ X > 0;\ \tau_0 = \varphi(\mathbf{R}, T),\ X = 0,\ T > 0. \tag{7}$$

As an additional condition we require the solution to be bounded when $T \to \infty$.*

We introduce a system of eigenfunctions $\psi_n(\mathbf{R})$ with eigenvalues ν_n satisfying the equation

$$\Delta_R \psi_n(\mathbf{R}) = -\nu_n^2 \psi_n(\mathbf{R}) \tag{8}$$

and the boundary condition (6), and also the expansions

$$\tau_0 = \sum_{n=0}^{\infty} f_n \psi_n,\ \mu = \sum_{n=0}^{\infty} M_n \psi_n,\ \psi = \sum_{n=0}^{\infty} \Phi_n \psi_n, \tag{9}$$

where $f_n = f_n(X, T)$; $M_n = M_n(X)$ and $\Phi_n = \Phi_n(T)$. The form of the eigenfunctions is determined by the cross-sectional geometry of the channel and the symmetry of the problem (i.e., the symmetry of functions μ and φ). In particular, in the plane problem ψ_n are expressed in terms of trigonometric functions, and in the axisymmetric case they are expressed in terms of Bessel functions. Explicit expressions for ψ_n and ν_n in these cases, and formulas for determination of the coefficients M_n and Φ_n in (9) can be found in [11], for instance.

For functions f_n we obtain from (4) and (7)-(9) the problem

$$\frac{\partial f_n}{\partial X} + \frac{\partial f_n}{\partial T} = \frac{\partial^2 f_n}{\partial T^2} - \Lambda \nu_n^2 f_n,\ f_n < \infty,\ T \to \infty,$$

$$f_n = M_n,\ T = 0,\ X > 0;\ f_n = \Phi_n,\ X = 0,\ T > 0. \tag{10}$$

In view of its linearity this problem can naturally be split into two parts, in one of which $\Phi_n = 0$, and in the other $M_n = 0$, describing, respectively, the effects of the initial temperature distribution and perturbations of the temperature in the channel entrance section on the temperature field. In this general case the solution of (10) can be expressed as the sum of the solutions of these two special problems.

For solution of the first problem it is convenient to use the Laplace transformation for X. Its solution in images will then take the form

$$\hat{f}_n = \hat{M}_n \exp\left[-T\left(\sqrt{p + \Lambda \nu_n^2 + \frac{1}{4}} - \frac{1}{2}\right)\right]. \tag{11}$$

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*On the solution of the initial system (1) we must impose initial conditions for the two mean temperatures τ_0 and τ_1. The approximate problem for (2) or (4) when these two conditions are imposed, however, does not in the general case satisfy the requirement of decay of the temperature perturbations with time and only one of them must be used. For instance, if at the initial time $\tau_0 = \tau_1 = 0$, then from the approximate problem for Eq. (4) with initial condition $\tau_0 = 0$ we obtain a solution in which the value of τ_1, given by relation (5), is non-zero at $T = 0$. This discrepancy, however is quite insignificant in the region of applicability of the approximate solutions, i.e., when $T \gg 1$.

For solution of the second problem it is convenient also in several cases to use the Laplace transformation for X and then solve the problem

$$\frac{d^2\hat{f}_n}{dT^2}-\frac{d\hat{f}_n}{dT}-(p+\Lambda\nu_n^2)\hat{f}_n=-\Phi_n(T),$$
$$\hat{f}_n=0,\ T=0;\ \hat{f}_n<\infty,\ T\to\infty, \tag{12}$$

directly. Sometimes it is more convenient to use the Fourier sine transformation for the variable T. We then obtain the solution in the form

$$f_n=\frac{2}{\pi}\exp\left(\frac{T}{2}\right)\int_0^\infty d\omega\exp\left[-X\left(\omega^2+\Lambda\nu_n^2+\frac{1}{4}\right)\right]\sin\omega T\int_0^\infty dz\,\Phi(z)\exp\left(-\frac{z}{2}\right)\sin\omega z. \tag{13}$$

Thus, the obtention of the solutions of the two special problems reduces to determination of the original of expression (11) or to the solution of (12) followed by determination of the original or to calculation of the integrals in (13). As a result we obtain a solution of the initial problem in the form of a series in (9), each term of which is the sum of the solutions of the indicated special problems with a given n.

We consider first the "washing out" of the initial temperature profile by the flow of working fluid, which at the entrance to the channel has zero temperature (which corresponds to $\Phi_n=0$, $n=1,2,\ldots$). For an arbitrary exponential initial temperature we have $M_n=A_n\exp(-a_nX)$, $\hat{M}_n=A_n(p+a_n)^{-1}$ and then from (11)

$$f_n(X,\ T)=\sigma_n^{(1)}(X,\ T;\ a_n)=\frac{A_n}{2}\exp\left(\frac{T}{2}-a_nX\right)\left\{\exp\left(-T\sqrt{\Lambda\nu_n^2-a_n+\frac{1}{4}}\right)\operatorname{erfc}\left(\frac{T}{2\sqrt{X}}-\sqrt{X\left(\Lambda\nu_n^2-a_n+\frac{1}{4}\right)}\right)+\exp\left(T\sqrt{\Lambda\nu_n^2-a_n+\frac{1}{4}}\right)\operatorname{erfc}\left(\frac{T}{2\sqrt{X}}+\sqrt{X\left(\Lambda\nu_n^2-a_n+\frac{1}{4}\right)}\right)\right\}. \tag{14}$$

It is obvious that $a_n=0$ corresponds to the case of a uniform profile. In addition, if the distribution $M_n(X)$ can be represented as the Laplace transform of some function $S_n^{(1)}(z)$:

$$M_n(X)=\int_0^\infty\exp(-zX)S_n^{(1)}(z)\,dz,\ \hat{M}_n=\int_0^\infty\frac{S_n^{(1)}(z)}{p+z}dz, \tag{15}$$

the dynamics of deformation of the initial temperature profile will be characterized by the following integral:

$$f_n(X,\ T)=\int_0^\infty S_n^{(1)}(z)\,\sigma_n^{(1)}(X,\ T;\ z)\,dz, \tag{16}$$

where function $\sigma_n^{(1)}$ id defined in (14). The conditions for validity of (16) reduce, in fact, to the conditions for existence of the integrals in (15) and (16), i.e., Eq. (16) is of a very universal nature.

The presented relations are simplified for a thermally insulated channel [in (6) $h=0$] with a uniform initial cross-sectional temperature distribution. In this case instead of series (9) for τ_0 we have expression (14) or (16), in which we put $\nu_n=0$.

We now consider the establishment of the temperature field generated by a flow at prescribed temperature entering the channel with zero initial temperature ($M_n=0$, $n=1,2,\ldots$). When the temperature at the entrance varies in accordance with an arbitrary exponential law, $\Phi_n=A_n\exp(-a_nT)$. After the solution of (12) and conversion to the original, we obtain

$$f_n(X,\ T)=\sigma_n^{(2)}(X,\ T;\ a_n)=A_n\exp[-X(\Lambda\nu_n^2-a_n(1+a_n))]\times\left\{\exp(-a_nT)-\frac{1}{2}\exp\left(\frac{T}{2}\right)\left[\exp\left(-T\sqrt{a_n(1+a_n)+\frac{1}{4}}\right)\operatorname{erfc}\left(\frac{T}{2\sqrt{X}}-\sqrt{X\left(a_n(1+a_n)+\frac{1}{4}\right)}\right)+\exp\left(T\sqrt{a_n(1+a_n)+\frac{1}{4}}\right)\operatorname{erfc}\left(\frac{T}{2\sqrt{X}}+\sqrt{X\left(a_n(1+a_n)+\frac{1}{4}\right)}\right)\right]\right\}. \tag{17}$$

$a_n=0$ corresponds to a constant temperature at the entrance. It is easy to obtain analogs of Eqs. (15) and (16). We have

$$\Phi_n(T)=\int_0^\infty \exp(-zT)S_n^{(2)}(z)\,dz,\; f_n(X,\ T)=\int_0^\infty S_n^{(2)}(z)\,\sigma^{(2)}(X,\ T;\ z)\,dz, \tag{18}$$

where $\sigma_n^{(2)}$ is defined in (17). For a thermally insulated channel and a uniform temperature distribution at the entrance, τ_0 is given by expressions (17) and (18) with $\nu_n=0$.

Of considerable interest (especially from the viewpoint of regulation of heat transfer [1, 2]) is the response of the system to a single temperature pulse at the channel entrance. Putting $\Phi_n(T)=A_n\delta(T-T_0)$, we obtain from (13) the impulse response function in the form

$$f_n(X,\ T;\ T_0)=\frac{A_n}{2\sqrt{\pi X}}\exp\left[-\frac{T-T_0}{2}-X\left(\Lambda\nu_n^2+\frac{1}{4}\right)\right]\left\{\exp\left[-\frac{(T-T_0)^2}{4X}\right]-\exp\left[-\frac{(T+T_0)^2}{4X}\right]\right\}. \tag{19}$$

This function is invariant relative to a shift of T_0, which is due to the effect of the initial condition at $T=0$ on the structure of the solution of the parabolic equation in (10). This solution has physical sense, of course, only when $T-T_0\gg 1$, i.e., in the calculations we must use the following response function:

$$f_n(X,\ T)=f_n(X,\ T;\ 0)=\frac{A_n}{2\sqrt{\pi X}}\exp\left[-\frac{T}{2}-X\left(\Lambda\nu_n^2+\frac{1}{4}\right)-\frac{T^2}{4X}\right]. \tag{20}$$

Using standard methods and the formalism of Green's function or the theorem of convolutions for integral transforms, we can express the response of the system to an arbitrary perturbation of the flow temperature at the channel entrance in the form of an integral, whose integrand contains the quantity (20).

If in (6) $h\neq 0$, then even in the case of a single uniform, i.e., independent of **R**, pulse at the entrance section the series (9) for τ_0 will contain terms corresponding to different n, and $A_n\neq 1$ are determined as the coefficients of expansion of the unit function in terms of a system of eigenfunctions $\psi_n(\mathbf{R})$. When $h=0$, however, the response of τ_0 to a pulse at time $T=0$ is expressed by Eq. (20) with $\nu_n=0$, $A_n=1$.

In applications it is important to know also the variation of the temperature of the walls of the heated channel and the heat flow through the wall, which has been considered earlier, in [12, 13], for instance. The wall bemperature can be regarded in a first approximation as equal to τ_0 when $\mathbf{R}\in C$; the value of $\partial\tau_0/\partial R_n$ when $\mathbf{R}\in C$, which determines the rate of heat exchange with the surroundings, can then be determined directly from condition (6).

The presented solutions enable us to consider a wide circle of very diverse problems of two-phase heat transfer in packed channels. The considered approximate formulation of the problem can easily be extended to more complex processes in which the flow of fluid is variable, the heat loss from the walls is nonuniform or depends on time, there are internal heat sources whose intensity in the general case depends on the temperature or on the concentration of some impurity in the flow, for which the corresponding mass-transfer problem must be jointly considered, etc. As an example, we investigate the operation of a periodically operating straight-through heat regenerator, which is of great independent interest. Possible approximate models of heat regenerators and analytical results for the simplest models have been given by Nusselt [14].

We consider a regenerator in which there is an alternating flow of hot (temperature τ_*) and cold (zero temperature) fluid. For simplicity, we assume that the velocity and physical parameters of the fluid in the heating and cooling periods, and also the lengths of these periods, are the same. Extension to the more general case is simple. If the initial temperature is zero, i.e., is the same as that of the surroundings, the course of heating is determined by the first series in (9); for the images of the coefficients fn of this series we have problem (12), in which $\Phi_n=\tau_*A_n$, where A_n are the coefficients of expansion of the unit function in terms of the complete system of eigenfunctions $\psi_n(\mathbf{R})$. The cooling of the flow during the first cooling period is characterized by expression (11), in which $\hat{M}_n$ is equal to $\hat{f}_n$, as it was at the end of the first heating period. In a similar way it is easy to investigate problems relating to subsequent heating and cooling periods.

Of particular importance is the steady periodic operation of the regenerator, where the temperature distributions at the same times for each of the heating and cooling periods are the same. We will assume that the temperature distribution established at the end of the cooling period is such that the coefficients of its expansion in terms of functions $\psi_n(\mathbf{R})$ are equal to $g_n^-(X)$, while at the end of the heating period the temperature distribution is characterized by the coefficients $g_n^+(X)$. In this case these quantities play the role of M_n in (10) and (11).

During the next heating period we have, from the results obtained above,

$$\hat{f}_n^+(T) = \hat{g}_n^- \exp\left[-T\left(\sqrt{p+\Lambda\nu_n^2+\frac{1}{4}}-\frac{1}{2}\right)\right] + \frac{A_n}{p}\left\{1-\exp\left[-T\left(\sqrt{p+\Lambda\nu_n^2+\frac{1}{4}}-\frac{1}{2}\right)\right]\right\} \tag{21}$$

(the time is measured from the start of the heating period). During the next cooling period

$$\hat{f}_n^-(T) = \hat{g}_n^+ \exp\left[-T\left(\sqrt{p+\Lambda\nu_n^2+\frac{1}{4}}-\frac{1}{2}\right)\right] \tag{22}$$

(the time is measured from the start of the cooling period).

The condition for establishment of the regime, i.e., the condition of stationarity of the limiting cycles, reduces to fulfilment of the equalities

$$\hat{f}_n^+(\Delta T) = \hat{g}_n^+, \quad \hat{f}_n^-(\Delta T) = \hat{g}_n^-, \tag{23}$$

where ΔT is the duration of one heating or cooling period. Solving Eqs. (21)-(23) and using the formula for the sum of an infinite geometric progression, we obtain

$$\hat{g}_n^+ = \frac{A_n}{p}\left\{1-\exp\left[-\Delta T\left(\sqrt{p+\Lambda\nu_n^2+\frac{1}{4}}-\frac{1}{2}\right)\right]\right\}$$
$$\times\sum_{m=0}^{\infty}\exp\left[-2m\Delta T\left(\sqrt{p+\Lambda\nu_n^2+\frac{1}{4}}-\frac{1}{2}\right)\right],$$
$$\hat{g}_n^- = \hat{g}_n^+\exp\left[-\Delta T\left(\sqrt{p+\Lambda\nu_n^2+\frac{1}{4}}-\frac{1}{2}\right)\right]. \tag{24}$$

Converting to originals, we obtain

$$g_n^+(X) = \frac{A_n}{2}\sum_{m=0}^{\infty}\exp(m\Delta T)\left\{\exp\left(-2m\Delta T\sqrt{\Lambda\nu_n^2+\frac{1}{4}}\right)\right.$$
$$\times\operatorname{erfc}\left(\frac{m\Delta T}{\sqrt{X}}-\sqrt{X\left(\Lambda\nu_n^2+\frac{1}{4}\right)}\right)+\exp\left(2m\Delta T\sqrt{\Lambda\nu_n^2+\frac{1}{4}}\right)$$
$$\times\operatorname{erfc}\left(\frac{m\Delta T}{\sqrt{X}}+\sqrt{X\left(\Lambda\nu_n^2+\frac{1}{4}\right)}\right)-\exp\left(\frac{\Delta T}{2}\right)\times$$
$$\times\left[\exp\left(-(2m+1)\Delta T\sqrt{\Lambda\nu_n^2+\frac{1}{4}}\right)\operatorname{erfc}\left(\frac{(2m+1)\Delta T}{2\sqrt{X}}\right.\right.$$
$$\left.-\sqrt{X\left(\Lambda\nu_n^2+\frac{1}{4}\right)}\right)+\exp\left((2m+1)\Delta T\sqrt{\Lambda\nu_n^2+\frac{1}{4}}\right)$$
$$\left.\left.\times\operatorname{erfc}\left(\frac{(2m+1)\Delta T}{2\sqrt{X}}+\sqrt{X\left(\Lambda\nu_n^2+\frac{1}{4}\right)}\right)\right]\right\}.$$

$$g_n^-(X) = \frac{A_n}{2}\sum_{m=0}^{\infty}\exp\left(\frac{(2m+1)\Delta T}{2}\right)\left\{\exp\left(-(2m+1)\Delta T\right.\right.$$
$$\left.\times\sqrt{\Lambda\nu_n^2+\frac{1}{4}}\right)\operatorname{erfc}\left(\frac{(2m+1)\Delta T}{2\sqrt{X}}\sqrt{X\left(\Lambda\nu_n^2+\frac{1}{4}\right)}\right)$$
$$+\exp\left((2m+1)\Delta T\sqrt{\Lambda\nu_n^2+\frac{1}{4}}\right)\operatorname{erfc}\left(\frac{(2m+1)\Delta T}{2\sqrt{X}}\right.$$
$$\left.+\sqrt{X\left(\Lambda\nu_n^2+\frac{1}{4}\right)}\right)-\exp\left(\frac{\Delta T}{2}\right)\left[\exp\left(-2(m+1)\Delta T\right.\right.$$
$$\left.\times\sqrt{\Lambda\nu_n^2+\frac{1}{4}}\right)\operatorname{erfc}\left(\frac{(m+1)\Delta T}{\sqrt{X}}-\sqrt{X\left(\Lambda\nu_n^2+\frac{1}{4}\right)}\right)$$

$$+\exp\left(2(m+1)\Delta T\sqrt{\Lambda\nu_n^2+\frac{1}{4}}\right)\operatorname{erfc}\left(\frac{(m+1)\Delta T}{\sqrt{X}}+\sqrt{X\left(\Lambda\nu_n^2+\frac{1}{4}\right)}\right)\Bigg]\Bigg\}. \quad (25)$$

The temperature distributions at the end and beginning of the heating period in the established regime are, respectively,

$$\tau^+(X,\ \mathbf{R})=\sum_{n=0}^{\infty}g_n^+(X)\psi_n(\mathbf{R}),\quad \tau^-(X,\ \mathbf{R})=\sum_{n=0}^{\infty}g_n^-(X)\psi_n(\mathbf{R}). \quad (26)$$

If the regenerator is thermally insulated, τ^+ and τ^- are the same as the functions in (25), with $\nu_n=0$. It is also easy to write explicit expressions for the temperature distribution at different times and to use them to evaluate the quantities characterizing the rate of heat transfer between the hot and cold flows, heat loss through the wall, etc., in relation to the flow velocity, duration of the heating and cooling periods, and length of regenerator, after which it is easy to optimize the regenerator with respect to these parameters. In view of their cumbersome nature the corresponding calculations are not given here.

Despite the complex form of relations (25), (26), etc., they are expressed in terms of known tabulated functions and do not require the laborious calculations involved in numerical evaluation of the integrals arising in the solution of the similar problem for system (1). In addition, the series in (25) converge fairly rapidly. It can be shown that the approximate model considered here allows some simplification of the analysis of generators of other types, e.g., countercurrent regenerators.

In a similar way, approximate solutions of the two problems of two-phase heat transfer, given above, have a much simpler form than the solutions of the corresponding problem based on system (1). Hence, we can expect considerable simplifications in the analysis of other more complex heat-transfer problems in apparatuses containing packings, especially in cases where solution (1) cannot be obtained at all for some reason or other. The validity of the solutions of (2) or (4) when $T \gg 1$ is convincingly demonstrated in [9], and also follows from the general analysis in [10].

NOTATION

a_n, A_n, coefficients; C, cross-sectional contour of channel; c, heat capacity per unit mass; d, density; f_n, coefficients of expansion of τ_0 in terms of eigenfunctions $\psi_n(\mathbf{R})$; g_n^-, g_n^+, values of f_n at start and end of heating period in steady operation of regenerator; h, coefficient in condition (6); L, characteristic linear scale of channel cross section; M_n, coefficients of expansion of function μ; p, Laplace transform parameter; **r**, **R**, dimensional and dimensionless transverse coordinates; $S^{(i)}$, functions in (15), (16), and (18); t, T, dimensional and dimensionless time; u, flow velocity calculated for total channel cross section; x, X, dimensional and dimensionless longitudinal coordinates; α_t, α_r, α_x, time scale and length scales in transverse and longitudinal directions; β, interphase heat-transfer coefficients; γ, parameter in (3); ΔT, duration of heating and cooling periods; ε, mean free cross section of channel or porosity of granular packing; λ, Λ, dimensional and dimensionless transverse thermal conductivity; μ, initial temperature distribution; ν_n, eigenvalues; $\sigma^{(i)}$, functions in (14) and (17); τ, mean temperature; $\tau*$, temperature of hot flow at entrance to channel; τ^-, τ^+, temperature distributions at start and end of heating period in steady operation of regenerator; φ, Φ_n, temperature profile at entrance to channel and coefficients of its expansion in terms of $\psi_n(\mathbf{R})$; ψ_n, eigenfunctions; ω, frequency; The subscripts zero and unity denote, respectively, the fluid and the solid packing; Λ indicates the Laplace transform.

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NONSTEADY FILTRATION OF SATURATED WATER VAPOR IN DISPERSE MEDIUM

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Analytic relations are obtained for the calculation of the temperature and pressure distribution in a disperse medium, and also the depth of the heated region in the filtration of saturated water vapor.

Questions associated with the investigation of saturated-water-vapor filtration in disperse media is of particular urgency at present in connection with the prospect of making effective use of the method of vapor-heat treatment as a means of increasing the petroleum yield of a bed. The main aim of the investigation, of course, is to obtain analytical relations allowing the vapor parameters in the course of filtration and its penetration depth in the plate to be obtained.

In most works devoted to the solution of this problem (e.g., [1-4]), integrodifferential heat-balance equations are used. However, in ignoring the hydrodynamics of the process, this approach can obviously only give satisfactory approximation in thermal calculations for very small ΔT, since it is assumed, in the absence of information on the form of the pressure or temperature distribution, that $\Delta T = \text{const}$.

In [5-8], an attempt was made to use relations obtained on the basis of a system of differential equations [9]. However, these equations were derived for the drying of capillary-porous bodies, and cannot be applied outside the scope of problems of diffusional-filtration transfer at small pressure gradients.

The physical picture of the problem is reflected more completely and accurately in [10], where a system of equations of nonisothermal multicomponent filtration is given. However, its use involves serious mathematical difficulties, and moreover mathematical inaccuracy was assumed in deriving the energy equation of the multicomponent flux.

Thus, as far as is known, relations for the calculation of saturated-vapor filtration in a disperse medium which are both sufficiently well-founded and expedient for use are not to be found in the literature at present.

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